## Nonlinear Psychometric Thresholds for Physics and Mathematics

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We analyze 5 years of student records at the University of Oregon to estimate the probability of success (as defined by superior undergraduate record; sufficient for admission to graduate school) in Physics and Mathematics as a function of SAT-M score. We find evidence of a nonlinear threshold: below SAT-M score of roughly 600, the probability of success is very low. Interestingly, no similar threshold exists in other majors, such as Sociology, History, English or Biology, whether on SAT combined, SAT-R or SAT-M. Our findings have significant implications for the demographic makeup of graduate populations in mathematically intensive subjects, given the current distribution of SAT-M scores.

A recent study (1) of 2000-2004 student records at the University of Oregon (UO) showed correlations of 0.35 to 0.5 between SAT scores and in-major, upper division GPA (i.e., 300-400 level courses in the student's major). These correlations are higher than those usually reported for SAT and freshman GPA (2,3) most likely because self-sorting of freshmen into more or less challenging courses weakens correlations with ability measures. UO has a number of favorable characteristics for this kind of study: it admits students with a broad range of abilities (middle 50% SAT-M + SAT-R: 990 - 1220, so little restriction of range), and the student body is relatively homogeneous (thereby minimizing the impact of ethnicity).

Despite the somewhat higher correlations, we discovered impressive cases of high achievement by students with relatively low SAT scores: in almost all majors (e.g., English, History, Sociology, Biology, etc.) students with combined (math + reading) scores well below 1000 (i.e., below the average among all SAT-takers) achieved in-major, upper division GPAs (henceforth, upper GPAs) in excess of 3.5 and even 4.0 (see figure 1 for a scatter plot of upper division GPA in History and Sociology versus SAT-Reading). These examples of the noisiness inherent in the SAT as a predictor of college performance could be interpreted as cases where other factors, such as extremely high conscientiousness or hard work, compensated for cognitive limitations or limitations in college preparedness. Overachievement of this type was found when comparing upper GPA to SAT combined, or SAT-M or SAT-R individually. Indeed, the simplest mathematical model one might formulate for GPA would include an ability input (presumably normally distributed, and correlated with SAT) and another input reflecting the effort exerted on the part of the student (this is plausibly normally distributed; in our university-wide data the fluctuations that remain after SAT is controlled for are roughly Gaussian with fairly constant standard deviation).

However, two majors stood out as qualitatively different from the others. In the cases of Physics and of pure Mathematics (defined by a set of rigorous courses taken by graduate school

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bound majors), the pattern of upper GPA versus SAT-M showed a sharp threshold: no student below approximately SAT-M = 600 was able to attain the 3.5 upper GPA (i.e., roughly equal numbers of A and B grades) typically required for admission to a Ph.D. program. (See figure 1, which displays upper GPAs vs SAT-M for Physics and Mathematics graduates in our data set.) A small fraction of students with SAT-M score at or above 600 attained upper GPA > 3.5; it seems plausible that these were the most conscientious and determined of students at this ability level. The fraction of students with high upper GPA increases monotonically with SAT-M. Note that upper GPA > 3.5 is not a high bar – the average of all upper division grades in these departments is about  $3.2 \pm .6$ , so 3.5 is only half a standard deviation above the average. The selection of GPA of 3.5 as a minimum value for admission to graduate school is, of course, somewhat arbitrary, but reflects our experience both with admissions to our graduate program in Physics and with the subsequent career paths of undergraduate majors at Oregon.

The existence of a minimum threshold, measurable by standardized tests, required for success in Mathematics and Physics has numerous important consequences. We expect that similar results also apply to highly mathematical fields of study such as some areas engineering or informatics.

- 1. If one regards SAT-M as simply a diagnostic, the score of 600 becomes a useful measure of readiness for college work in these fields. Indeed, our data suggests that the probability of success below this level is very low (see analysis to follow). Students who test below this level and who intend to major in a mathematically intensive subject should be offered enrichment in the basics of high school level mathematics before continuing on to college level work in these fields. Programs meant to increase the presence of underrepresented groups in STEM fields should focus on basic skills perhaps a gap year after high school which focuses on raising SAT-M related capabilities would prove successful. Many universities offer intensive summer preparation programs in basic college skills for incoming students from underrepresented groups. It is possible that prospective STEM majors with low SAT-M scores would benefit from similar programs specifically focused on math skills.
- 2. If one regards the SAT as a measure of general cognitive ability (4,5), then the 600 minimum gives an indication of the intrinsic difficulty level of the university curriculum in Physics and Mathematics. As such it has important psychometric implications. SAT-M 600 is roughly 75th percentile among test takers, and, depending on assumptions concerning the SAT-taking population relative to the whole, roughly 85th percentile in the general population. Thus, a strong interpretation of our result would be that even the most determined student is unlikely to master undergraduate Physics or Mathematics if their quantitative ability is below 85th percentile in the overall population. To have a 50 percent or greater chance of success (i.e., for a person of average conscientiousness or work ethic), one needs SAT-M well above 700, or in the top few percent of the overall population.

Under *either* interpretation 1. or 2., a minimum threshold of 600 has important implications. Because 600 is the *lowest* score at which even the most determined students can succeed, the bulk

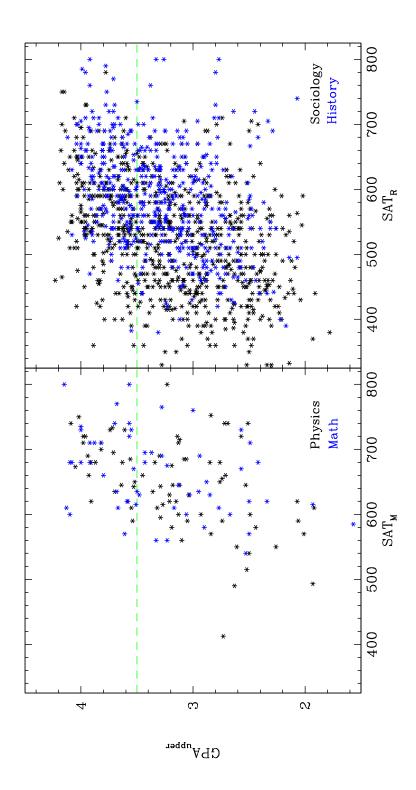


Fig. 1.— Modest evidence for a cognitive threshold is found in the SAT Math scores of Physics and Mathematics majors (left panel), but is not seen in SAT (Reading or otherwise) scores for majors such as Sociology and History (right panel). The green line indicates GPA of 3.5.

of students in Ph.D. programs in these fields will tend to have SAT-M scores of 650 or even 700+. This estimate is consistent with average GRE-Q scores well above 700 for applicants to programs in physics, math, engineering and computer science (see supplementary materials of Ceci, Williams & Barnett 2009 (6), or http://www.ets.org/Media/Tests/GRE/pdf/gre\_0910\_guide.pdf). In our UO data, same student SAT-M and GRE-Q scores exhibit a correlation of 0.75 and best-fit linear slope of close to unity. Therefore, the average graduate school applicant in these fields likely scored well above 700 on SAT-M. At SAT-M score of 650, the ratio of white males to females is almost 2:1, the percentage ratio of white males to African-Americans is roughly 10:1, and the percentage ratio of Asian-Americans to white males is almost 2:1. These ratios become even more extreme at higher scores – see College Board data on percentile equivalents by group at http://professionals.collegeboard.com/profdownload/SAT-Percentile-Ranks-by-Gender-Ethnicity-2009.pdf.

Below we present evidence supporting the existence of a minimum threshold at SAT-M roughly 600. Note, our analysis is agnostic as to possibilities 1. and 2. above. The interpretation of the data in figure 1 suffers from the relatively small numbers of students with low SAT-M. It is not hard to understand this paucity: students with low SAT-M typically perform poorly in Physics and Mathematics, and are unlikely to persist in the major through graduation. Our records show that many students begin the physics sequence, but later switch majors. In the case of Mathematics, there are several tracks through the major and only a small fraction of students attempt the rigorous courses (necessary for graduate study) we used to calculate upper GPA. The freedom of students to choose their major and to choose their curriculum makes it difficult to study the implications of low SAT-M for success in these fields.

To remedy this problem we consider student records during the same 2000-2004 period for the year long calculus-based physics course (251-252-253, henceforth 25X; UO is on the quarter system) taken by all Physics majors and many Mathematics and Chemistry majors. 25X is not an upper division course, but it has a standard and relatively rigorous syllabus. We have verified that the "style" of the course during this period was relatively consistent. In total, we have 826 terms of grade records for this course, and the associated SAT-M scores of the students. The distribution of average grades in 25X is shown in figure 2. Note an almost identical threshold, again at SAT-M just below 600.

We further analyze the 25X data set as follows. Divide the students into bins by SAT-M score, and estimate for each bin the probability p that an individual student earns an A-type (A+, A or A-) grade during particular term. (Bin sizes were varied in an effort to keep similar total numbers in each bin, although this was not possible for the lowest and highest scoring bins.) The simplest estimate of p is obtained by computing the actual fraction of A grades for students in the bin. However, we can do better than this using the cumulative binomial distribution: we determine the largest p which is, at 95 percent confidence level, consistent with the number of A grades actually earned. These p values are displayed in figure 4 (red curve). Note the increase in p with SAT-M score.

## PHYS 25X

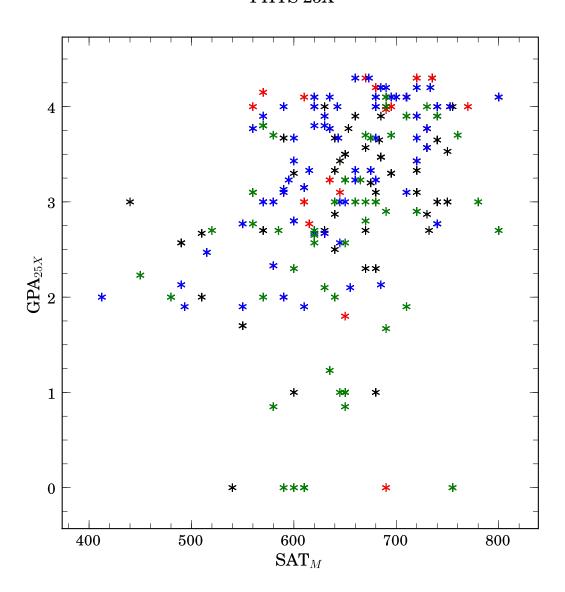


Fig. 2.— SAT Math scores versus average GPA in PHYS 25X series of courses (physics with calculus). Blue symbols are Physics majors, red are Math majors, black are other majors, green are students who did not graduate from the University.

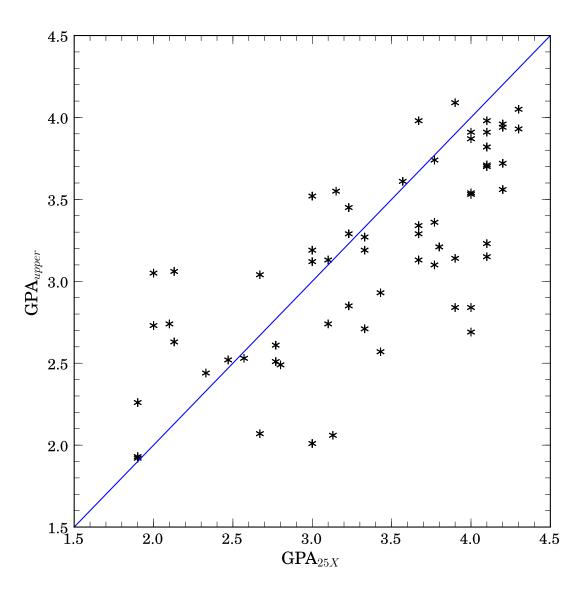


Fig. 3.— The correlation between grades in PHYS 25X and upper division Physics courses (300 or greater). The blue line is equality. The correlation coefficient is 0.75.

We observe that upper division (300 and 400 level courses) are typically much more challenging than 25X. Indeed, student grades in 25X correlate 0.75 with upper division GPA, with slightly higher grades earned, on average, in 25X (see figure 3). Therefore, the probability  $p_u$  of an A-type grade in upper division courses is similar to but likely smaller than the p obtained above for 25X. We now derive an upper bound on the probability P(GPA > 3.5) that a student in a particular SAT-M bin can earn a cumulative upper GPA greater than 3.5. The typical graduate school bound Physics major takes at least 16 upper division terms of advanced Physics and Mathematics courses (again, we assume the quarter system; another benefit of the Oregon data is the larger number of grade records compared to another school on the semester system). To exceed upper GPA of 3.5 would typically require at least 8 A-type grades amongst these 16 terms (note A+ grades are relatively rare compared to A or A- grades). Assuming  $p > p_u$ , we can deduce an upper bound on P(GPA > 3.5), assuming that at least 8 A grades of some sort are required out of 16 courses in order to obtain an average upper GPA of 3.5:  $P(GPA > 3.5) \approx P(\geq 8 \text{ A's} | 16)$ . The latter probability can be calculated using  $p_u$  and the binomial distribution. Results are shown in the final column in figure 4: blue points are upper bounds on P(GPA > 3.5) and the red curve is p at 95 percent confidence level. We obtain a threshold at around SAT-M 600: the probability of success (sufficient for graduate school admission) is negligible at 95 percent confidence level for SAT-M less than 600. For scores above 600 the success probabilities in the table are increasingly large, but recall these are only upper bounds, for several reasons: we are using the largest single term success probability p that is compatible with data at 95 percent confidence; the calculation essentially assumes that all non-A grades are some form of B, which is optimistic, etc.

The error bars (figure 4) on the upper bound on P(GPA > 3.5) are obtained using a Monte Carlo technique as follows. In each SAT-M score bin, we randomly remove half the data points and recompute p and P(GPA > 3.5). The errors shown indicate the variation in mean SAT-M score in each bin (horizontal error bar) and in P(GPA > 3.5) (vertical error bar), averaging over 100 such randomizations. We have checked that these average variations are stable after 100 randomizations.

The upper bounds obtained in figure 4 are fully compatible with the actual upper GPA data presented in figure 1, although note that even at very high SAT-M score (e.g., well above 700) the actual observed fraction of students who earn GPA > 3.5 never approaches unity. To have a fifty percent chance of GPA > 3.5 probably requires SAT-M score well above 700 (i.e., in the top few percent of the college bound population). It is plausible that for a person of average conscientiousness or work effort to have a high probability of performing well enough to be admitted to graduate school in Physics or Mathematics requires math ability or readiness near the top percentile.

We can give a simple, intuitive, explanation for the existence of the observed threshold. While the probability of success (i.e., earning an A grade) in any *single term* of Physics or Mathematics is smoothly varying and nearly linear with SAT-M (see red curve in figure 4), the probability  $P(\geq 8 \text{ A's} | 16)$  of earning 8 or more such A grades out of 16 courses is highly non-linear in SAT-M. Once the single term success probability p falls below a threshold, the cumulative GPA probability

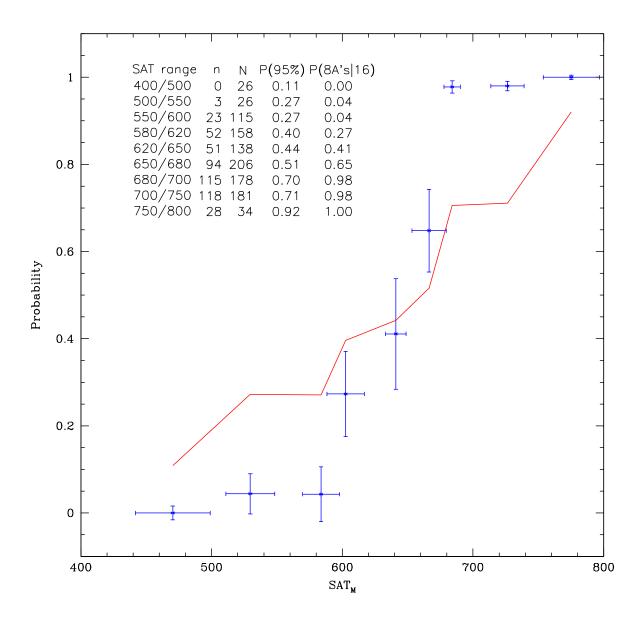


Fig. 4.— Probability charts for SAT Math versus grades in Physics classes. For each SAT bin, the number of A-type grades (n) is listed along with the total number of grades per bin (N). The red line displays the 95% probability P(95%) and the blue symbols display an upper bound on the probability that a student will achieve 8 or more A's out of 16 courses (typically necessary for an upper division GPA of 3.5.)

 $P(\geq 8 \text{ A's} | 16)$  drops to zero very fast. The non-linearity is generated by the cumulative nature of the undergraduate grade record.

Throughout this discussion we have been agnostic to the underlying cause of the SAT-M score-GPA correlation. Under assumption 2. the SAT-M cut-off of 600 reflects general cognitive ability or g, which would require very intensive intervention to elevate, in contrast to 1. which takes no position on the underlying ability issue. Ultimately, this is an empirical issue in need of actual data on the effect of various interventions to elevate low-scoring students' performance.

## References

- 1. Hsu, Stephen D. H. and Schombert, James (2010). Data Mining the University: College GPA Predictions from Sat Scores (April, 14 2010). Available at SSRN: http://ssrn.com/abstract=1589792
- 2. Sackett, Paul R., Borneman, Matthew J. & Connelly, Brian S. American Psychologist. High Stakes Testing in Higher Education & Employment, Vol 63(4), May-Jun 2008, 215-227.
- 3. Berry, C. M. & Sackett, P. R. (2009). Individual differences in course choice result in underestimation of college admissions system validity. Psychological Science, 20, 822-830.
- 4. Park, G., Lubinski, D., and Benbow, C. P. (2008). Ability differences among people who have commensurate degrees matter for scientific creativity. Psychological Science, 19, 957-961.
- 5. Lubinski, D., and Benbow, C. P. (2006). Study of Mathematically Precocious Youth after 35 years: Uncovering antecedents for the development of math-science expertise. Perspectives on Psychological Science, 1, 316-345.
- 6. Ceci, S. J., Williams, W.M., & Barnett, S.M. (2009). Women's underrepresentation in science: Sociocultural and biological considerations. Psychological Bulletin, 135, 218-261.